# Channel On Demand: Optimal Capacity for Cooperative Multi-channel Multi-interface Wireless Mesh Networks 

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#### Abstract

Cooperative communication (CC) has been proposed recently as an effective way to mitigate channel impairments. It has been shown that cooperative communications have the potential to significantly increase the capacity of wireless mesh networks (WMNs). However, most of the works focus on single channel based WMNs. In this paper, we will demonstrate how the cooperative communication benefits multi-channel multiinterface WMNs. The two strategies, amplify-and-forward (AF) and decode-and-forward (DF), have been widely used in cooperative communication to enhance the network capacity. We propose a novel mathematical model called channel-on-demand (COD), which combines the two strategies, together with direct transmission (DT). COD investigates the maximum capacity to accumulate the network resources, and provides the optimal interface assignment for real-time flows in cooperative multichannel multi-interface wireless mesh networks (CM2WMNs). COD analyzes maximum capacity for each node with both unidirectional flows and bidirectional flows. Based on the analytical results of COD, we provide four rules to apply analytical results. We also evaluate the proposed algorithm and compare it with simulation results using NS-2. Through simulation results, we show the significant rate gains achieved in CM2WMNs.

Index Terms-amplify-and-forward, cooperative communication, capacity, decode-and-forward, multi-channel multiinterface, wireless mesh networks (WMNs).


## I. Introduction

With the advantage of broadcast in wireless mediums, cooperative communication (CC) has been proposed and widely used recently [1]-[3]. In a wireless cooperative communication system, each user is assumed to transmit data and acts as a cooperative agent for another user. That is to say, each user transmits both its own bits as well as some information for its partner. Most previous work on CC is based on the single channel mesh networks [1], [4]-[6]. However, multichannel based wireless mesh network can enhance the network capacity more than the single channel based networks.

Figure 1 shows four cases of communication flows in cooperative multi-channel multi-interface wireless mesh networks (CM2WMNs), a special type of wireless mesh networks (WMNs), where the nodes are equipped with multiple channels. In directed graph, Bayesian network [7] are directed acyclic graphs. It have been proved that there are three possible types of adjacent triplets allowed in a directed acyclic graph.


Fig. 1. Four typical deployments with CM2WMNs.

Accordingly, in this paper, we provide the following four cases (an additional case for bidirectional links). Figures 1 (a), (b), and (c) show the relay node with single unidirectional flows and multi-unidirectional flows, respectively. We use dashed lines for the cooperative transmission. Figure 1 (d) shows the relay node with bidirectional flows. As we can see from Figure 1 (b), there are two flows in this network: $S_{1} \rightarrow D$ and $S_{2} \rightarrow D$. The relay node $R$ could use some of the interfaces for flow $S_{1} \rightarrow D$. The remaining interfaces could be used for $S_{2} \rightarrow D$. In this way, both of links $S_{1} \rightarrow D$ and $S_{2} \rightarrow D$ could enhance their capacity.
In this paper, we propose a novel on-demand service called channel on demand (COD). The proposed mechanism will combine three transmission strategies: amplify-and-forward (AF), decode-and-forward (DF), and direct transmission (DT). The amplify-and-forward method is to use the relaying capability of partners to achieve higher throughput [8]. The decode-and-forward method is to exploit the wireless broadcast advantage. Also, we will use direct transmission, which offers the largest capacity compared with the other two strategies. These three methods can be jointly used in CM2WMNs and achieve high capacity.
Based on the different metric of capacity, we develop a mathematical model for interface assignment, which could be used to obtain the maximum capacity for real-time traffic. This model includes two parts: optimal capacity for the interface
assignment with the unidirectional flows, and optimal capacity for direct node and relay node assignment with bidirectional flows. Based on the two parts, we provide the mark and unmark rules for relay node (i.e. selecting and unselecting relay nodes), and decomposition and priority rule to apply the analytical results. We also evaluate our results in NS-2. We offer the simulation results of throughput, end-to-end delay, and packet loss rate. We summarize the contributions of this paper as follows:
(1) We develop a mathematical model which analyzes the maximum capacity achieved by a single node for unidirectional and bidirectional traffic in CM2WMNs, presented in Figure 1.
(2) We provide the mark rule for the relay node and the unmark rule to remove the invalid relay node. Also, we use a decomposition method to divide the network into subgraphs, and the priority rule to obtain the maximum capacity of each subgraph, efficiently.
(3) We also evaluate our analytical results with simulation results. It shows that the proposed COD can achieve more throughput, lower end-to-end delay, and packet loss rate.

The remainder of this paper is summarized as follows: Section II gives a brief overview of the related work. Section III demonstrates the problem formulation of the system. Section IV presents the conditions for our relay node assignment. Section V provides the mathematical analysis model to obtain the maximum capacity. We also provide four rules to apply the analytical results. Section VI gives a discussion of the simulation results. This paper concludes in Section VII.

## II. Related Work

The classical relay channel models a class of three-terminal communication channels originally introduced by Van Der Meulen in [9]. The capacities of wireless mesh networks (WMNs) are extensively investigated in literature by [10]. They establish the capacity of general multi-channel networks wherein the number of interfaces may be smaller than the number of channels.
J. N. Laneman et al. in [11] outlined several strategies employed by the cooperating radios, including fixed relaying schemes, such as amplify-and-forward, decode-and-forward, and selection relaying schemes that adapt based upon channel measurements between the cooperating terminals, and incremental relaying schemes.
S. Sharma et al. in [5] proposed the joint optimization problem of relay node assignment and flow routing for concurrent sessions. They study this problem via mathematical modeling, and solve it using a solution procedure based on the branch-and-cut framework. However, this solution focused on single channel transmission for each direction.

In this paper, we use multi-channel multi-interface relay node assignment, which is based on the traffic demand. Our work is different from [5]. We do not consider the mesh point as the multi-hop relay because the capacity of the mesh node could be complicated due to a different topology, according to [12]. In our proposed scheme, the interfaces could be assigned
due to the traffic demand. Based on our proposed assignment, we can obtain the maximum capacity for the current traffic. Then, we formalize the problem in the next section.

## III. Problem Formulation

In this section, we define the model formulation for our algorithm in CM2WMNs. Table I shows the notation list of this paper, except Section VI.
(1) Due to the traffic demand information, we suppose that the number of interfaces assigned for each traffic flow should be different according to the different traffic demand. The expected traffic among them, the link capacities, and the interface assignment of the relay node, determines the route through the network for each communicating pair.
(2) The number of distinct channels that can be assigned to the node is bounded by the number of interfaces. The channels are working with the full duplex model, which allows communication in both directions, simultaneously.
(3) We do not consider the interference among the effect of the channel frequencies. This means we assume that all of the channels we use have no interference with each other. We configure the relay channel (either AF or DF), which is the same as the channel working for the direct transmission.
(4) We distinguish the relay node and direct node for different strategies. The relay node is the node which works as the cooperative partner. The direct node is the node working for the direct transmission.
(5) Instead of limiting ourselves to the time-slot model, we employ an orthogonal channel model for CC in WMNs. Usage of orthogonal channels has been widely accepted for cooperative communication.
(6) We do not offer the routing algorithm in our models. We simply assume the directions and paths of the flows are already known. The flow in this paper is the total traffic load for that directed link. Thus, for each link, we may have different traffic rates.

## IV. Conditions for the relay node assignment

In this part, we provide several conditions, which should be satisfied in our analytical model discussed in the next section. We classify these constrains as three parts: channel constraint, function constraint, and flow fairness.

## A. Channel constraint

Suppose that the node, say $i$, has $N_{i}$ interfaces which could be assigned with different channels. Each node could act as three functions: amplify-and-forward (AF), decode-andforward (DF), and direct transmission (DT). The channels assigned to each interface should be different. However, besides DT, a specific interface of one node could only select one function AF or DF. The existing tradeoff is that wireless resources are wasted since the relay node uses wireless resources to relay the signal from source to destination. Thus, in our model, we do not assign additional channels for AF and DF. AF and DF will work on the same channel as DT. Suppose that node $i$

TABLE I
Notation list

| Parameter | Description |
| :--- | :--- |
| $n_{d t}^{i}, n_{a f}^{i}, n_{d f}^{i}$ | Num. of interfaces of node $i$ with DT, AF, or DF |
| $n_{i}^{x}$ | Num. of interfaces for node $i$ on flow $x$ |
| $\beta_{d t}^{i}, \beta_{a f}^{i}, \beta_{d f}^{i}$ | Binary variable for the use of DT, AF, or DF |
| $\delta_{a f}^{i n}, \delta_{d f}^{i n}$ | Num. of interfaces selected for in flows as AF/DF |
| $\delta_{a f}^{o u t}, \delta_{d f}^{o u t}$ | Num. of interfaces selected for out flows as AF/DF |
| $f_{i}^{x}$ | Unidirectional traffic rate of flow $x$ on node $i$ |
| $C_{i}^{f_{i}^{x}}$ | Total capacity assigned for $f_{i}^{x}$ |
| $C_{d t}^{f_{i}^{x}}, C_{a f}^{f_{i}^{x}}, C_{d f}^{f_{i}^{x}}$ | The capacity of DT, AF, or DF on $f_{i}^{x}$ |
| $F_{i n}^{i}$ | Total num. of unidirectional flows on node $i$ |
| $F_{i j}^{r}$ | Total num. pairs for directional flows for the relay |
| $N_{i}$ | node $r$ |
| $f_{\overrightarrow{i j}}$ | Num. of available interfaces on node $i$ for unidi- |
| $N_{\overrightarrow{i j}}$ | rectional flows |
| $N_{\overline{i j}}$ | Traffic rate from node $i$ to node $j$ |
| $C_{i}$ | Num. of interfaces for flow from node $i$ to node $j$ |
| $\xi_{\overline{i j}}$ | Num. of interfaces for link of nodes $i$ and $j$ |
| $\epsilon_{i}$ | Capacity of node $i$ for unidirectional flows |
|  | Capacity between direct nodes $i$ and $j$ for bidirec- |
| tional flows |  |

has $N_{i}$ interfaces. Each interface of the node could use one of the two functions at most once: AF or DF. Then, we have:

$$
\begin{equation*}
n_{a f}^{i}+n_{d f}^{i} \leq n_{d t}^{i} \tag{1}
\end{equation*}
$$

where $n_{d t}^{i}, n_{a f}^{i}$, and $n_{d f}^{i}$ are the number of interfaces of node $i$ assigned for DT, AF, and DF, respectively. However, the total number of interfaces assigned for the flows should be limited to $N_{i}$ :

$$
\begin{equation*}
\sum_{k=1}^{F_{i n}^{i}} n_{d t}^{i}=N_{i}, \forall k \in \mathbb{N} \tag{2}
\end{equation*}
$$

## B. Functional constraint

Next, we introduce the functional constraint for cooperative communication. We use integer variables to identify whether this node could be used as AF or DF.

$$
\begin{align*}
& \beta_{a f}^{i}=\left\{\begin{array}{l}
1, \text { if the node } i \text { is used for AF } \\
0, \text { otherwise }
\end{array}\right.  \tag{3}\\
& \beta_{d f}^{i}=\left\{\begin{array}{l}
1, \text { if the node } i \text { is used for DF } \\
0, \text { otherwise }
\end{array}\right. \tag{4}
\end{align*}
$$

However, the direct node must be used for the communication. Although we have several channels assigned for different strategies, the relay node could only work as one function for the current pair due to signal-to-noise ratio (SNR) for the specific position. Then, we can use the following equation to formalize the constraint:

$$
\begin{equation*}
0 \leq \beta_{a f}^{i}+\beta_{d f}^{i} \leq 1 \tag{5}
\end{equation*}
$$



Fig. 2. The relay node in cooperation with a single unidirectional flow.

This equation means the relay node could be used as one of the functions for one flow.

## C. Flow rate fairness

Without special mechanisms in place, MAC unfairness in wireless networks not only can lead to unfair flow-level bandwidth allocation, but also lead to starvation of some flows. In this section, we study the flow fairness model suitable for our analysis. The assignment for each flow should follow the traffic demand. This means that if the traffic demand is higher, the total capacity arranged for this flow is also higher. If the capacity assigned to each interface could balance the traffics for each other, then, the arrangement will achieve the fairness bandwidth. According to the above requirement, we could use the following equations:

$$
\begin{equation*}
\frac{f_{i}^{1}}{C^{f_{i}^{1}}}=\frac{f_{i}^{2}}{C^{f_{i}^{2}}}=\ldots=\frac{f_{i}^{n}}{C^{f_{i}^{n}}} \tag{6}
\end{equation*}
$$

As shown in equation (6), $f_{i}^{x}$ is the traffic of flow $x$, which goes into node $i . C^{f_{i}^{x}}$ is the capacity assigned for the flow $x$ on node $i$. This means the traffic rate is proportional to the capacity.

## V. MAXIMUM CAPACITY FOR CM2WMNs

In this section, we present our optimal mathematical model for the interface assignment in CM2WMNs. This section will have two parts: interface assignment of relay node for unidirectional flows and bidirectional flows, separately. The first optimal model is the interface assignment used for single unidirectional flow, and multiple unidirectional flows. The second part of our optimal model is the interface assignment for bidirectional flows.

## A. Maximum capacity for unidirectional flows

In this part, we will first present the interface assignment of the relay node only used for single unidirectional flow. Then, we will discuss the relay node used for multiple unidirectional flows in the second part.

1) Maximum capacity of the relay node cooperating with single unidirectional flows: Figure 2 shows the example for this kind of transmission. As shown in Figure 2, the relay node $R$ is only used for one flow $S_{1} \rightarrow D_{1}$. The purpose of the proposed COD algorithm is to achieve the maximum bandwidth online. If the relay node $R$ does not have other flows, this node will only serve as AF, DF, or neither of them, as shown in Figure 2. For this purpose, we assign the


Fig. 3. The relay node in cooperation with multiple unidirectional flows.
same channels for the direct transmission $S_{1} \rightarrow M$, as for the cooperative transmission $S_{1} \rightarrow R \rightarrow M$.

In general, if we redefine this node $M$ as $i$, then, we have the following equation (7) to get the total capacity of node $i$ in regards to each flow $x$.

$$
\begin{equation*}
C^{f_{i}^{x}}=n_{d t}^{i}\left(C_{d t}^{f_{i}^{x}}+\beta_{a f}^{i} C_{a f}^{f_{i}^{x}}+\beta_{d f}^{i} C_{d f}^{f_{i}^{x}}\right) \tag{7}
\end{equation*}
$$

where $f_{i}^{x}$ is the traffic rate of flow $x$ on node $i$, and $C^{f_{i}^{x}}$ is the total capacity on the flow $x$ of node $i$. For node $i$, if we consider all of unidirectional flows, the objective function for the capacity of the node $i$ could be expressed as follows:

$$
\begin{equation*}
\max C_{i}=\sum_{x=1}^{F_{i n}^{i}} C^{f_{i}^{x}} \tag{8}
\end{equation*}
$$

subject to (1), (2), (3), (4), (5), (6).
Here, $F_{i n}^{i}$ is the total number of unidirectional flows on node $i$. It is not difficult to solve this integer linear program (ILP) problem. We should obtain the maximum capacity of node $i$ and number of interfaces for each flow. Since the channel working for the relay is the same as the direct node, the number of relay node interfaces should be limited to $N_{i}$. Thus, we can get the interface assignment for each node: $S_{1}$, $S_{2}$, and $R$.
2) Maximum capacity of the relay node cooperating with multiple unidirectional flows: Figure 3(a) shows a more complex model for multiple flow transmissions. In this case, the interfaces assigned for node $M$ are separated for different paths. There are two flows $S_{1} \rightarrow D_{1}$ and $S_{2} \rightarrow D_{2}$ in this example. The relay node $R$ could use AF or DF as the transmission strategy. For node $D_{2}$, it could be used for the direct transmission because it is a destination node for the $f_{m}^{2}$. However, as we can see from Figure 3(a), this node is also the relay node for the $f_{m}^{1}$. It is shown that node $M$ has two unidirectional flows: $S_{1} \rightarrow M$, and $S_{2} \rightarrow M$. Meanwhile, the relay node $R$ could help both of the flows: $f_{m}^{1}$ and $f_{m}^{2}$.

Figure 3(b) shows another example. The three nodes are responsible for unidirectional flows, which is the other direction compared with the first case. As we can see from Figure 3(b), the interface assignments of node $M$ and node $R$ are based on the number of interfaces and traffic demand. Therefore, the solution is the same as in Figure 3(a).

We redefine node $M$ as $i$ for the general case. Then, we formalize the capacity for each flow $x$ going into node $i$, as

```
Algorithm 1 Maximum capacity for unidirectional flows
    Input: \(f_{i}^{x}, C_{d t}^{i}, C_{a f}^{i}, C_{d f}^{i}, N_{i}\)
    for \(n_{d t}^{i}=1\) to \(N_{i}\) do
        for \(n_{a f}^{i}\) to \(N_{i}-n_{d t}^{i}\) do
                for \(n_{d f}^{i}\) to \(N_{i}-n_{d t}^{i}-n_{d f}^{i}\) do
            Get capacity for each flow from equation (9).
            Obtain total capacity value from equation (10).
            if The total capacity is larger than the current max-
            imum value record. then
                    Update the current maximum value.
                end if
            end for
        end for
    end for
```

follows:

$$
\begin{equation*}
C^{f_{i}^{x}}=n_{d t}^{i} C_{d t}^{f_{i}^{x}}+n_{a f}^{i} \beta_{a f}^{i} C_{a f}^{f_{i}^{x}}+n_{d f}^{i} \beta_{d f}^{i} C_{d f}^{f_{i}^{x}} \tag{9}
\end{equation*}
$$

Accordingly, the capacity of node $i$ could be divided by several links from different directions. Then, the total capacity of this node could be:

$$
\begin{equation*}
C_{i}=\sum_{i=1}^{F_{i n}^{i}} C^{f_{i}^{x}} \tag{10}
\end{equation*}
$$

To achieve the maximum capacity, we know the number of channels for each flow should be different according to the capacity and traffic demand. According to equation (6), we have the objective function below:

$$
\begin{equation*}
\max C_{i}=\sum_{x=1}^{F_{i n}^{i}} \frac{f_{i}^{x}}{f_{j}^{y}} C^{f_{i}^{y}}, \forall y \in F_{i n}^{i}, j \in \mathbb{N} \tag{11}
\end{equation*}
$$

subject to (1), (2), (3), (4), (5), (9)
As we can see from the above equation, we need to decide the assignment for both the number of interfaces and the transmission strategy arranged for each flow. The maximum capacity of node $M$ is according to the traffic demand and total interfaces of node $M$. Equation (11) is also an integer programming problem. To solve this problem, we propose an algorithm, shown in Algorithm 1. Then, we will discuss the complexity of our algorithm.

Theorem 1: Given the information of traffic rate and the number of interfaces, it will need $O\left(N_{i}^{3} F_{i n}^{i}\right)$ times to obtain the maximum capacity for each node.

Proof: As we can see from the above algorithm, we need three circulations for each strategy. Each of them is required to calculate the capacity of total unidirectional flows. The comparison for each flow and the selection of the transmission strategy will need $2 * F_{i n}^{i}$ times. Then, the total calculation times are $2 N_{i}^{3} F_{i n}^{i}$. Thus, the time complexity is $O\left(N_{i}^{3} F_{i n}^{i}\right)$.


Fig. 4. An example for unidirectional flows.
3) Examples for unidirectional flows: In this part, we show an example to explain our algorithm. We use Figure 4 to explain the example. There are 2 flows in this figure. The flow rates are 20 kbps and 30 kbps . Each node has 5 interfaces. Table II shows the detailed configuration. To obtain the optimal assignment for node $M$, we will first decide $\beta_{a f}^{m}$ and $\beta_{d f}^{m}$ for each flow. Then, we use equation (9) to obtain the value $C^{f_{m}^{1}}=n_{d t}^{m} \times 40+n_{d f}^{m} \times 30$ and $C_{m}^{f_{m}^{2}}=n_{d t}^{m} \times 40+n_{a f}^{m} \times 30$. To get the maximum capacity value and the interfaces assignment, we can use equation (11) and Algorithm 1. The results are presented in Table III. The maximum capacity of node $M$ is $C_{m}=320$.

TABLE II
CONFIGURATION FOR UNIDIRECTIONAL FLOWS.

| $N_{m}=5$ | $f_{m}^{x}$ | $C_{d t}^{m}$ | $C_{a f}^{m}$ | $C_{d f}^{m}$ |
| :--- | :--- | :--- | :--- | :--- |
| $S_{1} \rightarrow D_{1}$ | 20 | 40 | 20 | 30 |
| $S_{2} \rightarrow D_{2}$ | 30 | 40 | 30 | 20 |

TABLE III
RESULT FOR UNIDIRECTIONAL FLOWS.

| $N_{m}=5$ | $\beta_{a f}^{m}$ | $\beta_{d f}^{m}$ | $n^{f_{m}^{x}}$ | $C^{f_{m}^{x}}$ |
| :--- | :--- | :--- | :--- | :--- |
| $S_{1} \rightarrow D_{1}$ | 0 | 1 | 2 | 140 |
| $S_{2} \rightarrow D_{2}$ | 1 | 0 | 3 | 180 |

## B. Maximum capacity for bidirectional flows

The optimal model discussed in the previous section was designed in order to offer a solution for unidirectional flows. In this part, we will provide an optimal mathematical model to obtain the maximum capacity for each node (direct node and relay node) related to the bidirectional flows. We first provide an optimal model for direct nodes related to bidirectional flows. Then, we will discuss the optimal relay node assignment to achieve both fairness and maximum capacity. Figure 5 provides an example with bidirectional flows in CM2WMNs.

As we can see from this example, the interface assignment of relay node $R$ is related to the assignment of direct nodes $S$ and $A$. Thus, we need to first provide the optimal capacity of direct nodes $S$ and $A$.

1) Maximum capacity of the direct node cooperating with bidirectional flows: In this section, we discuss the maximum capacity for the direct nodes $S$ and $A$, shown in Figure 5.

As shown in the example, nodes $A$ and $S$ have assigned $n_{a}^{1}$ and $n_{s}^{2}$ interfaces for flow 1 and flow 2 from equation (11).


Fig. 5. Relay node in cooperation with bidirectional flows.

If the available common interfaces of nodes $A$ and $S N_{\overline{a s}}$ is sufficient for both of them, which is larger than $n_{a}^{1}+n_{s}^{2}$, then, we use the current strategy. However, if the current interfaces are not sufficient to satisfy the requirement, which is smaller than $n_{a}^{1}+n_{s}^{2}$, we need to adjust our choice. Thus, we need to discuss the lower bound of $N_{\overline{a s}}$ for direct nodes $S$ and $A$.

We know that direction transmission has the largest capacity compared with the other two transmission strategies. Thus, there is a trade-off between the diversity gain and the waste of the spectrum resource in cooperative diversity. Then, the lower bound of the interfaces $N_{\overline{a s}}$ should be with the condition of direct transmission. Next, we offer a theorem to prove this hypothesis:

Theorem 2: In a bidirectional transmission, the lower bound of $N_{\overline{a s}}$ is the case of direct transmission. That means $n_{a f}^{i}=0$ and $n_{d f}^{i}=0$.

Proof: To prove the theorem, we define $f_{s}^{1}, f_{s}^{2}, \ldots, f_{s}^{n}$ and $f_{a}^{1}, f_{a}^{2}, \ldots, f_{a}^{n}$ to be the flows for nodes $S$ and $A$. According to equation (6), we have:
$\frac{f_{s}^{1}}{C^{f_{s}^{1}}}=\frac{f_{s}^{2}}{C^{f_{s}^{2}}}=\ldots=\frac{f_{s}^{n}}{C_{f_{s}^{n}}^{n}}$ and $\frac{f_{a}^{1}}{C^{f_{a}^{1}}}=\frac{f_{a}^{2}}{C^{f_{a}^{2}}}=\ldots=\frac{f_{a}^{n}}{C^{f_{a}^{n}}}$
Suppose $f_{s}^{1}$ is the flow for $A \rightarrow S$ and $f_{a}^{2}$ is the flow for $S \rightarrow A$. Then, we have $C^{f_{s}^{1}}=\frac{f_{s}^{1} \sum_{x=2}^{n} C^{f_{s}^{x}}}{\sum_{x=2}^{n} f_{s}^{x}}$. The lower bound of $n_{a}^{1}+n_{s}^{2}$ is shown as follows:

$$
n_{a}^{1}+n_{s}^{2}=\frac{f_{s}^{1} \sum_{x=2}^{n} C^{f_{s}^{x}}}{\sum_{x=2}^{n} f_{s}^{x}}+\frac{f_{a}^{2} \sum_{x=1, x \neq 2}^{n} C^{f_{a}^{x}}}{\sum_{x=1, x \neq 2}^{n} f_{a}^{x}}
$$

As we all know that if the value of capacity is higher, more interfaces should be assigned for $n_{a}^{1}+n_{s}^{2}$. However, the direct communication must exist for the flows. So, the lower bound is the number of interfaces assigned only for direct transmission when $n_{a f}^{i}=0$ and $n_{d f}^{i}=0$.

As we mentioned above, when $n_{a}^{1}+n_{s}^{2} \leq N_{a s}$, we use equation (11) for the assignment. Otherwise, we need to adjust it. If we use general expression $N_{\vec{i} j}$ and $N_{\vec{j} i}$ for the number of interfaces assigned for bidirectional flows, we can formalize the following equation:

$$
\xi_{\overline{i j}}=\left\{\begin{array}{l}
\max C_{i}+\max C_{j}, N_{\overrightarrow{i j}}+N_{\overrightarrow{j i}}<N_{\overline{i j}}  \tag{12}\\
\max \left\{C_{i}+C_{j}\right\}, N_{\overrightarrow{i j}}+N_{\vec{j} i} \geq N_{\overline{i j}}
\end{array}\right.
$$

Then, we discuss the first case for equation (12). From equation (9), we have $C^{f_{i}^{x}}=n_{d t}^{i} C_{d t}^{f_{i}^{x}}$. According to equation (11), we have:

$$
\begin{equation*}
\xi_{\overline{i j}}=\max \sum_{x=1}^{F_{i n}^{i}} \frac{f_{i}^{x}}{f_{i}^{y}} C^{f_{i}^{x}}+\max \sum_{x=1}^{F_{i n}^{j}} \frac{f_{j}^{x}}{f_{j}^{y}} C^{f_{j}^{x}} \tag{13}
\end{equation*}
$$

In the second case of equation (12), if the number of interfaces $N_{a s}$ is larger than the lower bound, we will need to make the adjustment regarding the whole maximum capacity and flow fairness. Thus, we have the objective function:

$$
\begin{equation*}
\xi_{\overline{i j}}=\max \left(\sum_{x=1}^{F_{i n}^{i}} \frac{f_{i}^{x}}{f_{i}^{y}} C^{f_{i}^{x}}+\sum_{x=1}^{F_{i n}^{j}} \frac{f_{j}^{x}}{f_{j}^{y}} C^{f_{j}^{x}}\right) \tag{14}
\end{equation*}
$$

$$
\text { subject to }(1),(2),(3),(4),(5),(6)
$$

2) Maximum capacity of the relay node cooperating with bidirectional flows: In this part, we will discuss the maximum capacity for the relay node $R$ in Figure 5. As shown in the example, the relay node $R$ has to decide how to assign the interfaces and help the bidirectional flows to achieve the maximum capacity.

If we use $\delta_{a f}^{i n}, \delta_{a f}^{o u t}, \delta_{d f}^{i n}$, and $\delta_{d f}^{o u t}$ to identify the number of interfaces assigned for different directions of flows, then, the sum of them has to be limited to the number of interfaces for the relay node $i$ and the total interfaces of $N_{a s}$. Thus, we have the follow equation:
$\delta_{a f}^{i n}+\delta_{d f}^{i n}+\delta_{a f}^{o u t}+\delta_{d f}^{o u t} \leq \min \left\{\min \left\{N_{\overline{a s}}, N_{\overrightarrow{a b}}+N_{\overrightarrow{s a}}\right\}, N_{r}\right\}$
Thus, we need to consider two cases: If we have $N_{r} \geq$ $\min \left\{N_{\bar{a} s}, N_{\overrightarrow{a s}}+N_{\overrightarrow{s a}}\right\}$, then, we follow the same interface assignment as the direct transmission. Otherwise, we need to offer a new method to get the value.

Then, we discuss the relay node assignment which is smaller than the current one. If we use $N_{r}$ as the remaining interfaces of node $R$ for a relay purpose, then, we can get the following equations for the total capacity $\epsilon_{r}^{f_{s}^{x}}$ of node $R$ as a relay node:

$$
\begin{equation*}
\epsilon_{r}^{f_{s}^{x}}=\delta_{a f}^{i n} C_{a f}^{f_{s}^{x}}+\delta_{d f}^{i n} C_{d f}^{f_{s}^{x}}, \epsilon_{r}^{f_{a}^{y}}=\delta_{a f}^{o u t} C_{a f}^{f_{a}^{y}}+\delta_{d f}^{o u t} C_{d f}^{f_{a}^{x}} \tag{16}
\end{equation*}
$$

Then, the objective function to obtain the maximum capacity could be formalized as follows:

$$
\begin{equation*}
\max \epsilon_{r}=\sum_{i=1}^{F_{s a}^{r}} \epsilon_{r}^{f_{s}^{x}}+\epsilon_{r}^{f_{a}^{y}}, \forall i \in \mathbb{N} \tag{17}
\end{equation*}
$$

subject to (1), (2), (3), (4), (5), (6).
Algorithm 2 provides the solution of this equation.
3) Examples for bidirectional flows: In this part, we will use an example to explain the algorithm above. Each node in this example has 5 interfaces.

Figures 6(a) and 6(b) offer two different cases, which are correspondingly case 1 and case 2 in equation (12). For each link in this figure, we give the flow rate and number of interfaces in brackets. For the first case, the interfaces obtained from equation (13) is $N_{\vec{a} \vec{b}}$ and $N_{\overrightarrow{s a}}$. The sum of $N_{\vec{a} \vec{b}}$ and $N_{\vec{s} a}$ is equal to five interfaces for each node. We do not adjust this assignment for case 1 , however, in case 2 , we will need

```
Algorithm 2 Maximum capacity for relay node
    Input:
    for \(\delta_{a f}^{i n}=1\) to \(\min \left\{\min \left\{N_{\overrightarrow{a s}}, N_{\vec{a} \vec{s}}+N_{\overrightarrow{s a}}\right\}, N_{r}\right\}\) do
        for \(\delta_{a f}^{o u t}=1\) to \(\min \left\{\min \left\{N_{\overrightarrow{a s}}, N_{\overrightarrow{a s}}+N_{\overrightarrow{s a}}\right\}, N_{r}\right\}-\delta_{a f}^{i n}\) do
            obtain capacity for each bidirectional flow according
            to equation (16).
            if the total capacity is larger than the current maximum
            value record then
                    update the current maximum value record.
            else
                    continue
            end if
        end for
    end for
```



Fig. 6. Examples with bidirectional flows.
TABLE IV
LOWER BOUND FOR BIDIRECTIONAL FLOWS.

| Figure 6(a) | $f_{\overrightarrow{i j}}$ | $N_{\overrightarrow{i j}}$ | Figure 6(b) | $f_{\overrightarrow{i j}}$ | $N_{\overrightarrow{i j}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $B \rightarrow S$ | 30 | 2 | $B \rightarrow S$ | 30 | 2 |
| $A \rightarrow S$ | 50 | 3 | $A \rightarrow S$ | 50 | 3-1 |
| $C \rightarrow A$ | 80 | 3 | $C \rightarrow A$ | 30 | 2 |
| $S \rightarrow A$ | 30 | 2 | $S \rightarrow A$ | 80 | 3 |

to adjust the assignment according to equation (14). Table IV shows the result of Figure 6. The assignment of $N_{\overrightarrow{a s}}$ in Figure $6(\mathrm{~b})$ is with the original assignment (3) and adjustment ( -1 ).

Figure 6(c) provides the example of the relay node assignment. From this example, after the initial assignment is obtained from equation (13), the relay node $M$ only has 2 interfaces for the flows $S \rightarrow A$ and $A \rightarrow S$. According to equation (17), we can obtain the value $\epsilon_{r}$ and the interfaces assignment. Table V gives the flow settings (shown in Figure 6(c) ) and results of this example. The assignment of flow $A \rightarrow S$ is with the original assignment (2) and adjustment $(+1)$. The relay node offers one interface for the flow $A \rightarrow S$ and $S \rightarrow A$, separately, shown in the bracket of Table V.

## C. Rules and interface assignment

Based on the above discussion, we offer four rules to be operated for the network. In this part, we will first offer the

TABLE V
ASSIGNMENT FOR THE RELAY NODE.

| Figure 6(c) | $f_{\overrightarrow{i j}}$ | $N_{\overrightarrow{i j}}$ |
| :--- | :--- | :--- |
| $B \rightarrow S$ | 50 | 3 |
| $A \rightarrow S$ | 30 | 2 |
| $C \rightarrow A$ | 80 | 3 |
| $S \rightarrow A$ | 30 | 2 |


(c) Unmark invalid relay node

Fig. 7. Marking progress and interface assignment.
selection progress, also called mark rule, for the relay node. Then, we provide several unmark rules for the relay node. In all of our figures, the node is colored gray if it is used as a relay node. It is colored white for direct node purpose. We then offer the procedure for the whole process. We are using a simple example (shown in Figure 7) to illustrate our approach.

Mark rule for the relay node. The rule to mark the relay node is quite simple. That is, if the flows exist in node $R$ 's neighbors: $S$ and $D$, then, this node $R$ could be the relay node for flow $S$ to $D$.

First, we need to get all nodes' neighbor information. Then, we use a directed graph to represent all of the flows. As shown in Figure 7(a), the regular line represents the connectivity of the nodes. The directed line shows that the flows exist in this network. Figure 7(b) shows the result after the mark rule. The procedure starts from node 1 . For node 1 , the two neighbors, node 2 and node 3, have a flow. Then, node 1 has been marked as the relay node. The rest can be done in the same manner. In this example, nodes $1,2,3,4$, and 7 can be selected as the relay nodes. However, to achieve the maximum capacity, not all of these nodes are assigned for the relay function. This is because if we set the node as the direct node, this node could benefit more capacity than that working as a relay node.

Unmark rule for the relay node. (1) If there are two relay nodes for the same flow, we only select one that could offer more capacity for the network. In this example of Figure 7, to the nodes $1,2,3$, and 4 , we know that both nodes 1 and 4 could serve as the relay node for flow from node 3 to node 2 . Then, we select node 1 for the relay, since this node is idle. Then, we unmark node 4 . However, in some other cases, node 4 may not be obvious to remove. Then, we need to keep node 4 , until we find the optimal one. It could either be node 1 or


Fig. 8. Results after the decomposition step 1.
node 4.
(2) If there is cyclic flows among the nodes, then, these nodes could also be the relay nodes of each other. As shown in Figure 7, nodes 2, 3, and 4 make a circle. Nodes 2, 3, and 4 could be relay nodes of flows $3 \rightarrow 4,2 \rightarrow 4$, and $3 \rightarrow 2$, respectively. Then, in this case, we only select one of them which could provide the most capacity. None of them could be selected if the benefit capacity of each node in the circle is the same, or else all the interfaces of the nodes have to be used as the direct transmission. The other nodes in this circle should be unmarked. However, as we have removed node 4 from the previous part, we still need to mark nodes 2 and 3 as relay nodes. In the following part, we will provide the solution to this problem.

We then offer a decomposition rule to divide the nodes into different subgraphs. From Figure 1, we have offered four typical network deployments when there is a relay node, which could help the direct nodes.

Decomposition rule. To apply the results from Figure 7, we need to decompose the nodes into the relationship sets, which could conform one of the four deployments. If we use relationship sets $\mathbb{A}, \mathbb{B}, \mathbb{C}$, and $\mathbb{D}$ to represent the four typical deployments in Figure 1, then, we have $\mathbb{A}=\{S, R, D\}, \mathbb{B}=$ $\left\{S_{1}, S_{2}, R, D\right\}, \mathbb{C}=\left\{D_{1}, D_{2}, R, S\right\}$, and $\mathbb{D}=\{S, R, D\}$. We construct the subgraph of Figure 7 only based on the four basic deployments, since these four cases are all possible types in acyclic directed graph. However, not all of the nodes are needed to construct the subgraph: First, if the node does not have any flows, we do not need to get the subgraph of that node. This is just the case of node 1 in our example; Second, if the node is only used for one flow, and the related link has been included in the other nodes' subgraph, we do not need to construct the subgraph for this node. As shown in Figure 7, nodes 7,8 , and 10 do not need the subgraph, since the related link has been included in the subgraph of node 9 .

Since not all of the flows have a relay node, the number of


(c)




Fig. 9. The results after the decomposition.
relay nodes could be zero. However, the direct transmission must follow the four basic deployments.

Then, we will start from the first node. The procedure could be operated as follows:
(1) For each subgraph, we will first partition the node's graph according to our four basic deployments. For example, node 3 has three flows $3 \rightarrow 2,3 \rightarrow 6$, and $4 \rightarrow 3$. In this step, node 3 should have 2 subgraphs $2 \leftarrow 3 \rightarrow 6$ and $4 \rightarrow 3$. Figure 8 offers the results after step 1 .
(2) We will remove the subgraph, which is completely included in other nodes' subgraph. In our example, the subgraph $4 \rightarrow 3$ is included in node 4's subgraph $3 \leftarrow 4 \rightarrow 5$, so we need to remove that graph. Figure 9 offers the final results after the decomposition.
(3) Interface assignment for each group: We need to calculate the maximum capacity and interface assignment of each group using the equation discussed in the previous section.
(4) Minimum interfaces for the same node in different subgraphs: If the same node exists in several subgraphs, and the node works as a direct node in one subgraph or the other subgraph, then, we need to get the sum of the interfaces for unidirectional links. If it is larger than the number of interfaces the node has, we need to adjust the assignment. Since we are using full duplex mode, we do not combine the results with the bidirectional links.

As shown in this example, we need to combine the results of assignments $4 \rightarrow 5$ and $6 \rightarrow 5$. However, we do not need to combine the results of $3 \rightarrow 6$ and $4 \rightarrow 3$, because the two links for node 3 are for different directions.
(5) Minimum interfaces for the same node as the relay node and direct node in different groups: The capacity of the direct transmission is always larger than that of the relay function. Therefore, if the number of interfaces assigned to the direct node is larger than the sum of interfaces assigned to the relay node, we will use the assignment of the direct node. However, even if it is smaller than the interfaces for the relay function, we will first use the assignment of the direct node, and then, use the remaining interfaces for the relay part according to different traffic demands.

As shown in Figure 9, node 3 is the direct node of (b), (c), and (e). It is also a relay node for (d). Because we are using full duplex mode, interfaces of nodes for the direct mode is the maximum value of the sum of the two directions: $\max \left\{N_{\overrightarrow{36}}+\right.$


Fig. 10. A three-node model in a CC system.
$\left.N_{\overrightarrow{32}}, N_{\overrightarrow{43}}\right\}$. So, the remaining number of interfaces for node 3 is $N_{3}-\max \left\{N_{\overrightarrow{36}}+N_{\overrightarrow{32}}, N_{\overrightarrow{43}}\right\}$. If the result is larger than 0 , then, this part of the interfaces could be used as a relay function for $2 \rightarrow 4$.

Priority rule. Based on the decomposition rule, we know that the same nodes could belong to several groups. Then, we define that the more popular node should be with the highest priority. The node is viewed as the most popular node when it exists in the most subgraphs as the direct node. Therefore, this node will be assigned first. In the example of Figure 9, nodes 5 and 6 are the most popular nodes. Among the subgraphs of nodes 5 and 6 , we select the subgraph when nodes 5 and 6 are in the middle. Figures 9(f) and (g) should be assigned first. Then, we check the subgraph (b). After this, we need to remove nodes 5 and 6 from the node set and select the most popular node from the remaining node set. In this way, our algorithm could work efficiently.

## VI. Performance Evaluation

In this section, we present our numerical results and performance evaluation to demonstrate the capacity gains that can be achieved by our interface assignment and marking progress in CM2WMNs. We compare our results with the single channel single interface multi-hop networks. We will also provide results compared with the non-cooperative multichannel multi-interface wireless mesh networks (NM2WMNs). We will first provide the evaluation metric for the capacity calculations. Based on the metric, we provide the mathematical results and simulation results.

## A. Evaluation Metric

In this part, we will present the capacity calculations for the three transmission strategies: amplify-and-forward, decode-and-forward, and direction transmission. The ultimate goal of the relay assignment is to maximize the overall network capacity, or the number of bytes it can transport between the traffic aggregation devices within a unit of time.

We will use the following equation for the remaining part. Based on this, we discuss the capacity of each transmission strategy:

$$
\gamma_{x, y}=S N R_{x, y}\left|\alpha_{x, y}\right|^{2}
$$

where $S N R_{x, y}$ is the signal to noise ratio for the link $x \rightarrow y$. $\left|\alpha_{x, y}\right|^{2}$ is the fading coefficient function.

For each transmission strategy, the intuitive idea is that if the capacity is higher, this path should be assigned more packets at each time slot. Next, we demonstrate the achievable capacity developed in [11]. We consider both of the models: amplify-and-forward and decode-and-forward. Also, we adopt


Fig. 11. Node deployment for simulation setup.
the direct transmission capacity model. We use the well known three-node model, as shown in Figure 10.

1) Direct Transmission ( $D T$ ): To establish baseline performance under direct transmission, the maximum average mutual information for DT is between input and output.As shown in Figure 10, the pair of nodes $S$ and $D$ are using the direct transmission mode. $B$ is the bandwidth. Then, according to Shannon's channel capacity, we have:

$$
C_{s, d}^{d t}=B \log _{2}\left(1+\gamma_{s, d}\right)
$$

where $C_{s, d}^{d t}$ is the channel capacity for direct transmission between node $S$ and node $D$.
2) Amplify-and-Forward Transmission (AF): The amplify-and-forward protocol produces an equivalent one-input, twooutput complex Gaussian noise channel with different noise levels in the outputs. According to [11], the maximum capacity $C_{s, d}^{a f}$ for AF between nodes $S$ and $D$ should be represented as follows:

$$
\begin{array}{r}
C_{s, d}^{a f}=\frac{B}{2} \log _{2}\left(1+\gamma_{s, d}+f\left(\gamma_{s, r}, \gamma_{r, d}\right)\right) \\
\text { where } f(x, y)=\frac{x y}{x+y+1}
\end{array}
$$

3) Decode-and-Forward Transmission (DF): Also, we have the following equation to obtain the maximum capacity $C_{s, d}^{d f}$ for DF between nodes $S$ and $D$, when the node is working as the decode-and-forward mode:

$$
\begin{array}{r}
C_{s, d}^{d f}=\frac{1}{2} \min \left\{C_{s, r}^{d t}, B \log _{2}\left(1+g\left(\gamma_{s, d}, \gamma_{r, d}\right)\right)\right\} \\
\text { where } g(x, y)=x+y
\end{array}
$$

The scheduling decision could be related to the capacity of each function, the number of flows, and the number of interfaces. Then, we use $n_{1}, n_{2}$, and $n_{3}$ as the number of interfaces assigned to each transmission strategy: direct, relay, and decode, respectively. If $X, Y$, and $Z$ are the number of packets assigned for each path, then we should have the following equation to achieve the fair arrival time:

$$
\frac{X}{n_{1} C_{s, d}^{d t}}=\frac{Y}{n_{2} C_{s, d}^{a f}}=\frac{Z}{n_{3} C_{s, d}^{d f}}
$$



Fig. 12. Numerical results with relay node assignment for Fig. 11.

## B. Simulation Setup

In our simulation, we use $B=36 \mathrm{MHz}$ bandwidth for each channel. The transmission power is 0.0316 w . We assume the variance of noise is $10^{-10} \mathrm{w}$. For simplicity, we only consider the propagation between the source and destination. So, it is given by $\left|\alpha_{x, y}\right|=|x-y|^{-4}$, where $|x-y|$ is the distance between $x$ and $y$. We will use random topology with the different number of interfaces for our evaluation. Each node is equipped with the maximum of 5 interfaces. In our simulation, we use constant bit rate (CBR) with UDP traffic pattern for our evaluation, because it is the maximum bit rate that matters, not the average. CBR could be used to take advantage of the whole capacity. The deployment of the nodes is shown in Figure 11. The traffic rate is measured in kbps.

As shown in Figure 11, the 25 nodes are distributed in a $1,190 \times 400$ square meter area. The channel frequency is $2.4 G H z$. We use the network simulator (NS-2) for our evaluation. The traffic pattern is the same for all of our evaluations. There are four cases for our performance evaluation:

- COD-CC: We use our proposed algorithm COD with CC.
- COD-DT: This means, we use our COD without a relay node. We only adopt the direct transmission.
- Ran-CC: We use random interface assignment. This means we randomly assign the interfaces without consideration of capacity. But, we adopt the relay node to help with the transmission.
- Ran-DT: We use random interface assignment and do not use CC.


## C. Numerical Results and Simulation Results

The numerical results obtained from our algorithm is presented in Figure 12 for the example in Figure 11. The number related with the link is the number of interfaces that should be assigned, while the number around the relay node is the interface assignment for the link to benefit. The result is the case with five interfaces of each node.

Figure 13 shows the throughput results for the 25 -node deployment. We offer the throughput results regarding the number of interfaces. The throughput results we provide are


Fig. 13. Simulation results with total throughput.


Fig. 14. Simulation results with average end-to-end delay.


Fig. 15. Simulation results with average packet loss rate.
the sum of all the nodes in the network. Through these results, we can see that with a different number of interfaces, the throughput has increased for all of the cases. Compared with other cases, our proposed COD could get more throughput. When our algorithm has five interfaces, it could achieve about 390 Mbps for the whole network. The reason is that we use cooperative communication as well as the multi-channel multiinterface environment. More than that, our purposed method tries its best to obtain the network resources, and achieve the traffic rate fairness.

We also provide the evaluation results with end-to-end delay. These are the average results regarding all the nodes in the network. This advantage is also obvious from this evaluation. As we can see from Figure 14, when we use our algorithm with five interfaces, the end-to-end delay results could reach 0.075 seconds on average. Because the relay nodes could also help the transmission, the total number of packets for the direct transmission is smaller than the normal case.

The results of the packet loss rate, presented in Figure 15, are the case with average rate per node. The packet loss rate is also the lowest when compared with other cases. The lowest rate with five interfaces is only 68 kbps .

In summary, we believe that the proposed COD method could benefit the network capacity and achieve flow fairness. As shown in simulation results in terms of throughput, end-to-end delay, and packet loss rate, our COD method could achieve the best performance compared with other cases.

## VII. Conclusion

In this paper, we propose a new method, called channel on demand (COD) in cooperative multi-channel multi-interface wireless mesh networks (CM2WMNs). The proposed mechanism investigates a best effort to accumulate the network resources, and provides the optimal interface assignment for real-time flows. COD analyzes the maximum capacity based on the traffic demand. COD provides the optimal mathematical model to achieve the maximum capacity for the four basic cases in the network model, as shown in Figure 1. To apply the numeral results, we provide a mark rule for the relay node, and an unmark rule to remove the invalid relay node. We use a decomposition method to divide the network into subgraphs, and the priority rule for the operating process. We also provide the numeral results and simulation comparison in terms of throughput, end-to-end delay, and packet loss rate.

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## REFERENCES

[1] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," IEEE Communications Magazine, 2004.
[2] J. Laneman, G. Wornell, and D. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in Proc. of IEEE ISIT, 2001.
[3] T. Hunter and A. Nosratinia, "Diversity through coded cooperation," IEEE Transactions on Wireless Communications, 2006.
[4] P. Bahl, A. Adya, J. Padhye, and A. Walman, "Reconsidering wireless systems with multiple radios," ACM Sigcomm Computer Communication Review, 2004.
[5] S. Sharma, Y. Shi, Y. Hou, H. Sherali, and S. Kompella, "Cooperative communications in multi-hop wireless networks: Joint flow routing and relay node assignment," in Proc. of IEEE Infocom, 2010.
[6] H. Shan, W. Zhuang, and Z. Wang, "Distributed cooperative mac for multihop wireless networks," IEEE Communications Magazine, 2009.
[7] J. Pearl, Bayesian Networks: A Model of Self-Activated: Memory for Evidential Reasoning. Computer Science Department, University of California, 1985.
[8] J. Cai, X. Shen, J. Mark, and A. Alfa, "Semi-distributed user relaying algorithm for amplify-and-forward wireless relay networks," IEEE Transactions on Wireless Communications, 2008.
[9] E. Van Der Meulen, "Three-terminal communication channels," Advances in Applied Probability, 1971.
[10] P. Kyasanur and N. Vaidya, "Capacity of multi-channel wireless networks: impact of number of channels and interfaces," in Proc. of ACM Mobicom, 2005.
[11] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," IEEE Transactions on Information Theory, 2004.
[12] "Capacity of wireless mesh networks: Understanding single radio, dual radio and multi-radio wireless mesh networks," white paper, BelAir Networks, 2006.

